The Equations of Westminster Abbey

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Abstract

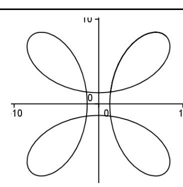
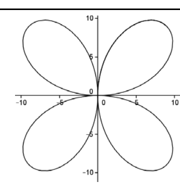
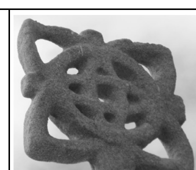
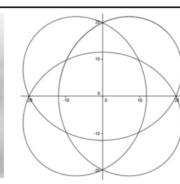
I have examined several geometric patterns of some stone monuments and constructed parametric equations that reconstruct their curves. This type of exercise may be a useful classroom activity for students or any other people interested in developing their experience with parametric equations. In particular, this essay is about the mathematical aspects of certain decorations found at Westminster Abbey and other examples.

Nowadays it is possible to teach parametric curves in a High School thanks to programmes such as Mathematica or Geogebra. Some instructions like *Manipulate* or *Slider* commands allow us to observe the behaviour of these curves just varying additional parameters.

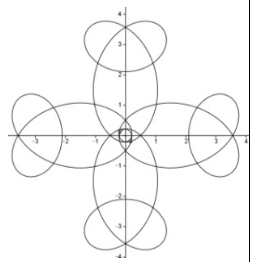
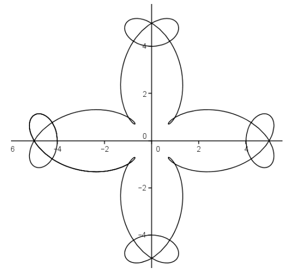
However, as the NCTM would say, which context can make this teaching attractive? In Bridges 2012, the Associated Professor Nils K. Rossing showed us that establishing the equations of rope rosettes can be an attractive context in two ways; he gets to the parametric equations of this design by studying their shape and development in Fourier series, but also by visualizing the relationship between their physical shape and the numbers that appear in the matching equations: (the number of bights equals the difference between the two rotational frequencies, and the number of crossings equals the frequency of the vector with the highest amplitude, and others) [1], [2].

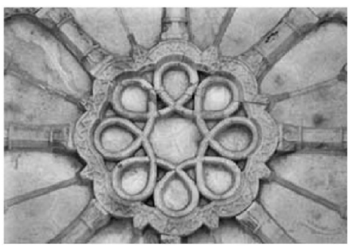
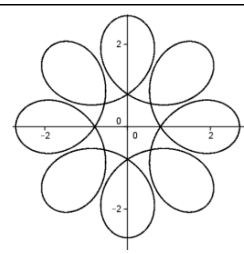
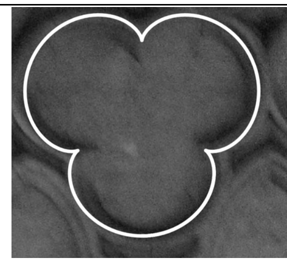
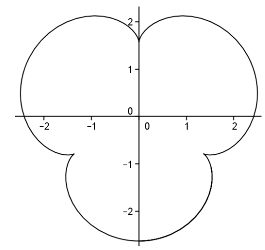
This encouraged me to investigate if I could apply this sight to any real figure – for example, a stone design – and if I could get to know its parametric equations, and then to know things like: Are the curves in the decoration of *The Campanile di Giotto* in Florence or of *Westminster Abbey* epicycloids or circles? The epicycloids whose Cartesian equations are (p cos(t) + cos(p t), p sin(t) + sin(p t)) where p-1, if p is integer, show the number of cusps of the epicycloid. Presenting the epicycloids and similar curves in this way would be more useful when using the above mentioned programmes, allowing us to assign an equation to many decorative curves in world wide architecture, for example, the ceiling of a church in Tarragona, the cathedral of León, or Westminster Abbey.

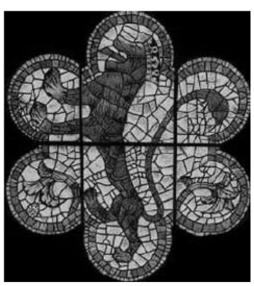
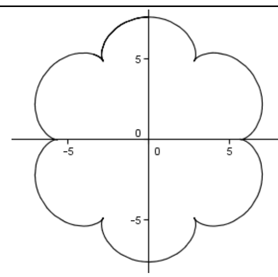
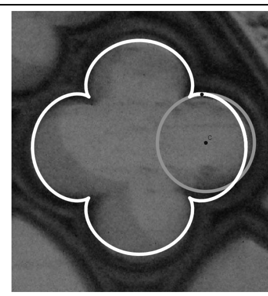
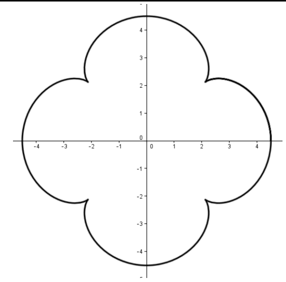
Below I am showing some examples which illustrate the result of my analysis. The equations are sometimes not accurate like in the Celtic crosses that you can see at the beginning.

 	 	 
San Martiño de Ferreira, Palas	*Kilmartin Cross, Argyll*	*Ancorados, A Estrada*
p=5	*p=6.34*	*p=18.25*
x=6.34 cos(-t + π / 4) + p cos(3t + π / 4); y=6.34 sin(-t + π / 4) + p sin(3t + π / 4) (This and all parametric equations in this paper for 0 < t < 2 π)		

Other type of Celtic cross, classic, has approximate equations that can be described with 3 terms. Moreover, you can find these equations in a certain three-dimensional knot. This cross has multiple variants and exact examples in Wales, Scotland and Ireland (Nevern Cross, Capel Gwladys Cross, Fahan Cross, Clonmacnoise,,..)

 	 
Santa Susana Cross, Santiago de Compostela x= 2.1 cos(t) + 1.15 cos(-3t) - 1.15 cos(9t) y= 2.1 sin(t) + 1.15 sin(-3t) - 1.15 sin(9t)	*One side of Nevern Cross in Wales* x= -3.5 cos(t) - 1.5 cos(-3t) + cos(9t) y= -3.5 sin(t) - 1.5 sin(-3t) + sin(9t)

 	 
Chuch of San Ramón of Tarragona x= 1.7 cos(t) + cos(7 t) y= 1.7 sin(t) - sin(7 t)	*Westminster Abbey* x= 4 cos(t + 3π / 2) + cos(4 t + 3π / 2) y= 4 sin(t + 3π / 2) + sin(4 t + 3π / 2)

 	 
León Cathedral x= 7 cos(t+π / 2) + cos(7 t+π / 2) y= 7 sin(t+π / 2) + sin(7 t+π / 2) Clearly, it's best rebuilt with circles.	*Westminster Abbey* x= 5 cos(t) + cos(5t) y= 5 sin(t) + sin(5t) The photo is not clear enough to distinguish between circles and epicycles.

References

[1] Nils Kr. Rossing, *The Old Art of Rope Work and Fourier Decomposition*, Bridges Towson 2012
[2] Nils Kr. Rossing and Christoph Kirfel, *Matematisk beskrivelse av taumatter* (2004)